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$c = 1$ matrix model from string field theory

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Abstract: We show that the boundary string field theory (BSFT) on unstable D0-branes in 2d string theory is equivalent to the double scaled $c = 1$ matrix model (i.e. quadratic action), even though we naively expect many interaction terms in BSFT. It is checked that S-matrices are trivial in the open string theory on the D0-branes. We discuss how to take the closed string decoupling limit in our model in order to make its holographic interpretation clear. We also find some useful lessons from our results in 2d string toward the further understanding of the higher dimensional BSFT action.

Keywords: String Field Theory, M(atrix) Theories, Tachyon Condensation, 2D Gravity.

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Contents

1. Introduction

It is important to obtain a non-perturbative and background independent formulation of the string theory in general backgrounds in order to understand some deep problems, for example, the resolution of the cosmological singularities in string theory. A significant progress in this direction is the BFSS matrix model proposal [1]. The action of the BFSS matrix model is the same as a low energy effective action on multiple D0-branes in type IIA superstring. However, this model is supposed to describe M-theory in an infinitely boosted flame. Another related problem is that a D0-brane in type IIA superstring has a conserved charge, i.e. D0-charge, so we can not construct D-branes without the D0-charge. This means that the D-branes always need to have nonzero field strengths and thus non-commutative world-volumes.

In order to overcome this difficulty, one might consider the matrix model based on non-BPS D0-branes or $\mathrm{D0} - \overline{\mathrm{D0}}$ pairs [2] since these branes have no conserved charges and it was shown that we can construct any D-branes from them [3, 2]. However, because of the tachyons in these unstable D0-branes we should employ a string field theory to analyze the dynamics of the unstable D0-branes. Since it includes infinitely many massive modes and derivatives, such a matrix model does not look tractable as it is.

Recently, it was proposed [4] that the $c = 1$ matrix model, which had been known to describe the two-dimensional (2d) string theory [5], can be considered as a tachyon action on multiple D0-branes. This matrix model description of 2d closed string theory is argued to come from a open-closed duality. Soon later, these D0-branes are identified with the so called ZZ-branes [6] in the boundary Liouville theory [7 − 9]. This idea was also successfully applied to the construction of a matrix model for the two dimensional type 0 string [10, 11].

Thus if this proposal is completely correct, we can seriously regard the $c = 1$ matrix model as a concrete realization of the idea of the matrix model based on unstable D0-branes. Even though it has been established that a fermion in the double scaled $c = 1$ matrix model is identified with a decaying unstable D0-brane, we only have a few evidences for the identification of the $c = 1$ matrix model with a tachyon action [12, 7]. Though we know that the perturbative tachyon mass agrees with that of the $c = 1$ matrix model [7], we are lacking precise understanding of the tachyon action of ZZ-branes including interactions.

Naively, it is naturally expected that there are non-trivial interaction terms as is true for string field theory actions in higher dimension. Then we immediately encounter a sharp paradox why the double scaled $c = 1$ matrix model is so simple that it does not include any interaction terms. One may think that taking the double scaling limit may save the situation since it might reduce the complicated tachyon action to the simplified quadratic action as suggested in [4]. However, this is problematic as long as we consider putting D0-branes in the conventional 2d string theory defined by the Liouville theory. This is because when we put an extra D0-brane as a probe, its action from the matrix model disagrees with the one from open string field theory.

We resolve this problem by considering the boundary string field theory (BSFT) action [13 − 19] on the ZZ-branes. The BSFT action for unstable D0-branes is originally a complicated one including infinitely many higher derivative terms. However, it can be greatly simplified via a field redefinition. Indeed, in the present paper we will argue that it is (classically) equivalent to the quadratic free action, which is exactly the same as the double scaled $c = 1$ matrix model.

We will give several independent evidences for this claim. First, we directly check that the on-shell four point interaction is vanishing in the open string theory on a ZZ-brane. Next, we extend this result to arbitrary interactions at any order of the perturbation theory by applying a slightly indirect approach of taking $c = 1$ limit of the non-minimal $c < 1$ string [20]. Finally, we move on to a direct examination of the field redefinition which relates the BSFT action and the simple quadratic action. We explicitly construct the field redefinition up to the cubic order in the BSFT action and check that it is indeed non-singular.

The organization of this paper is as follows. We first state our main claim that the BSFT action on unstable D0-branes in 2d string theory is classically equivalent to the double scaled $c = 1$ matrix model (i.e. quadratic action) in section 2. Then we give evidences for this claim in section 3 by analyzing the on-shell amplitudes and the BSFT action. In section 4 we discuss how to take the closed string decoupling limit in our model

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in order to make its holographic interpretation clear. In the section 5 we try to find some useful lessons from our results in 2d string toward the further understanding of the higher dimensional BSFT action. In section 6 we draw conclusions and discuss future problems.

2. Main claim: $c = 1$ matrix model from BSFT action

The main claim we would like to argue in this paper is that *at the the classical level (or tree level), the BSFT action for non-BPS N D0-branes (ZZ-branes in the Liouville theory) in two dimensional bosonic or type0 string is equivalent to the quadratic action*

$$S = \frac{1}{g_s}\int dt \mathrm{Tr}\left[\frac{1}{2}(D_t\Phi)^2 + \frac{1}{2}\Phi^2 - 1\right], \tag{2.1}$$

where Φ is a N by N hermitean matrix and $D_t = \partial_t - i[A_0,]$ is the covariant derivative of the $\mathrm{U}(N)$ gauge symmetry.[1] Of course, if we take a limit $N \to \infty$, this action is same as the double scaled $c = 1$ matrix model action except the constant energy shift. We can also obtain the BSFT action for N $\mathrm{D0}-\overline{\mathrm{D0}}$ branes just by replacing the hermitean matrix Φ and the gauge group $\mathrm{U}(N)$ with a complex matrix and $\mathrm{U}(N) \times \mathrm{U}(N)$, respectively. A closely related proposal for general string field theories in 2d string was also proposed in [12].

In other words, we can relate both theories via a non-singular field redefinition. At a higher loop level, we need to take the measure of path-integral into account and this issue is beyond the scope of the present paper. However, as we will discuss later in the next section, we have a strong evidence which supports the action (2.1) is correct even if we include all perturbative loop corrections.[2]

It is recently proposed that the $c = 1$ matrix model can be identified with the theory of unstable D0-branes and it describes the closed string theory via a kind of open-closed duality or holography [4]. In 2d string theory, an unstable D0-brane can be described by a ZZ-brane [6] in the Liouville CFT with the Neumann boundary condition in the time-direction [7]. The physical field on it is only the tachyon field T and the gauge field A_0. There is no transverse scalar field because the linear dilaton prevents any movements of the D-brane in the Liouville direction. Thus, the open string theory on the unstable D0-branes agrees with the $c = 1$ matrix model *when we neglect interactions*. To see if this idea does really make sense, we need to understand the reason why there are no interaction terms like Φ^3, Φ^4 etc. in the (double scaled) $c = 1$ matrix model (2.1).

Our claim, presented just before, not only supports this conjecture, but also shows that the interaction terms in BSFT can be set to zero via the field redefinition. Consider M D0-branes in 2d string theory. In the $c = 1$ matrix model description, we can regard them as M extra eigenvalues x_i $(i = 1, 2, \ldots, M)$ sitting above the Fermi level[3] $E = -\mu$. It is well-known that all eigenvalues in the matrix model behave as free fermions. When

[1] We have set $\alpha' = 1$ for bosonic string and $\alpha' = \frac{1}{2}$ for type 0 string.

[2] Clearly this agrees with what we expect from the $c = 1$ matrix model.

[3] In spite the energy shift in (2.1), below we define the energy for each eigen-value by the standard formula $H = \frac{1}{2}(p^2 - x^2)$. Here we have used a rescaled coordinate $x = \Phi\sqrt{g_s}$ and its conjugate momentum p.

there is no open string tachyon condensation on them $T = 0$, they are sitting at the top of the potential $x_i = 0$. Then the M D0-branes can be described by the matrix model (2.1) with the $M \times M$ matrix Φ. Since we cannot put a fermion below the Fermi level, we must require the important constraint

$$p^2 - x^2 \geq -2\mu\,. \tag{2.2}$$

On the other hand, when we consider the BSFT action for the M unstable D0-branes, our claim shows the classical equivalence between the $c = 1$ matrix model action and the BSFT action via a field redefinition.

The constraint (2.2) intuitively corresponds to the fact that the tachyon potential[4] in BSFT is bounded from below. In bosonic string, the potential is given by $V(T) = (T+1)e^{-T}$ [15]. In this case this potential is bounded for the positive T, while unbounded for the negative T. This nicely agrees with the $c = 1$ matrix model description of 2d bosonic string [5], where only one side is filled with the fermions. On the other hand, in the superstring case, the potential of non-BPS D0-branes and $\mathrm{D0}-\overline{\mathrm{D0}}$ branes are given by the gaussian profile $V(T) = e^{-T^2}$ [16] and $V(T, \bar{T}) = e^{-T\bar{T}}$ [17, 18], respectively. Since they are completely bounded from below, it again matches with the fermi surface structures in the type 0B and 0A matrix model [10, 11].

When the tachyon T is not a constant, its profile with $E(T, \dot{T}, \ldots) < -\mu$ at an instant should be excluded. If we compare the classical solutions in both sides, the constraint (2.2) in the $c = 1$ matrix model matches with that of the λ parameter $0 \leq \lambda \leq 1/2$ [22]. Note that the classical solutions of BSFT should be the same as those of BCFT or the boundary state because they all use the same boundary perturbation and are essentially the same for the on-shell structure. In this way, the claim is consistent with the open-closed duality interpretation of $c = 1$ matrix model in a highly nontrivial way. We will see more direct evidences in the next section.

3. Evidences for the Equivalence

In this section we will give important evidences for the claim stated in the previous section.

First we study on-shell amplitudes of open string tachyons in 2d string theory. It is very important to notice that the momenta of the on-shell tachyons are severely restricted from the energy-momentum conservation in one dimensional D-brane world volume. Actually, this requires that the in-state and out-state should be the same because there are only two on-shell states ($e^{X^0/\sqrt{\alpha'}}$ and $e^{-X^0/\sqrt{\alpha'}}$).

However, we still have the possibility that some nontrivial phase for the S-matrix elements exists like the scalar ϕ^4 theory. Below, we will argue that in the ZZ-brane case the S-matrix is indeed trivial, i.e. $S = 1$. We show this by an explicit computation of the four point interaction at tree level in section 3.1, and also by a slightly indirect approach at any order of the perturbation in section 3.2. Since any smooth field redefinition does

[4]Interestingly, the exponential potential in BSFT is directly related to the loop operator which creates another kind of D-brane (so called FZZT-brane) as noticed in [21]. The relation between this observation and our arguments in this paper is not obvious at present.

not change S-matrices, we expect they are trivial on the D0-branes in 2d string theory if their action is indeed equivalent to the quadratic one of the $c = 1$ matrix model.[5] It is noted here that that the BSFT is supposed to reproduce the perturbative S-matrix. This is because the BSFT action is reduced to the partition function when all fields are on-shell and it can be regarded as S-matrix generating effective action [23 – 25].

Next we study the BSFT action itself order by order with respect to the power of the tachyon field. We explicitly compute the field redefinition which relates the BSFT action to the $c = 1$ matrix model up to the cubic order and show that it is indeed non-singular in section 3.3.

3.1 Four tachyon on-shell scattering amplitude

Since the scattering amplitude for odd number tachyons are all zero due to the momentum conservation, the lowest non-trivial one is the four tachyon amplitude. Thus let us compute this and check that it is vanishing. The Liouville sector has no effect in this computation because only the time coordinate X^0 obeys the Neumann boundary condition on the D0-brane. Therefore we can just reduce the standard formula of S-matrices in $D = 26$ (or $D = 10$) dimensional (super)string to those in $D = 1$ dimension. For general dimension D, the well-known result for bosonic string (i.e. Veneziano amplitude, see e.g. [26]) is given by setting $\alpha' = 1$

$$\begin{aligned} S(k_1, k_2, k_3, k_4) = {} & 2ig_0^2(2\pi)^D\delta^D(\sum_i k_i) \times \\ & \times \left[\frac{\Gamma(-s-1)\Gamma(-t-1)}{\Gamma(-s-t-2)} + \frac{\Gamma(-t-1)\Gamma(-u-1)}{\Gamma(-t-u-2)} + \frac{\Gamma(-u-1)\Gamma(-s-1)}{\Gamma(-u-s-2)}\right], \end{aligned} \tag{3.1}$$

where we have defined the Mandelstam variables $s = -(k_1 + k_2)^2$, $t = -(k_1 + k_3)^2$ and $u = -(k_1 + k_4)^2$. The on-shell condition for a tachyon vertex operator e^{ikX} is $k^2 = 1$. The variables s, t and u are not independent and obey the constraint $s + t + u = -4$ in any dimension D. In the one dimensional case, which we are interested in, we can choose the values $k_1 = k_2 = 1$ (incoming) and $k_3 = k_4 = -1$ (outgoing), or equivalently $s = -4$ and $t = u = 0$. At this point, however, the amplitude (3.1) becomes singular. To regularize it, we first assume $D > 1$ and finally take the limit $D = 1$. We define the infinitesimal variable ϵ by $s = -4 + \epsilon$ and also assume t and u are infinitesimal of the same order. Then the sum of the Γ function ratios in (3.1) can be written as follows up to $O(\epsilon)$ terms[6]

$$\frac{\Gamma(-s-1)\Gamma(-t-1)}{\Gamma(-s-t-2)} + \cdots = -\frac{(t+u)(\epsilon+t+u)}{tu} = 0\,. \tag{3.2}$$

In the final equality we have used the constraint $s + t + u = -4$.

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[5] Strictly speaking, this S-matrix we consider is formal because we can not define the asymptotic field in one-dimensional "field theory". However, this formality of the discussion is sufficient for showing the equivalence because we do not want to consider the physical scattering amplitude.

[6] To show this we used the formula $\Gamma(-n+\epsilon) = \frac{(-1)^n}{n!}(\epsilon^{-1} + (\sum_{k=1}^n k^{-1}) - \gamma) + O(\epsilon)$ for a zero or positive integer n.

We can also do the similar computations[7] in the superstring (i.e. type 0 string) case and find the same conclusion.

This absence of the four tachyon interaction may be understood from the fact that the Veneziano amplitude is a sum of the contributions from the propagators of open string modes. Because there are no massive or massless modes on the ZZ-brane, it is natural that the four point interaction vanishes for the ZZ-brane.

Thus we have found that the four particle scattering amplitude in the open string theory is vanishing. Then it is natural to guess that all non-trivial on-shell scattering amplitudes are zero at tree level.

3.2 General on-shell amplitudes via $c < 1$ deformation

To avoid the singular behavior in the $c = 1$ string (=2d string), it is also helpful to assume an infinitesimal background linear dilaton in the time-direction in addition to the standard space-like linear dilaton. Actually this was the way how the closed string S-matrices were computed in 2d string theory [27]. Then the coupling constant behaves like $g_s = e^{qX^0+Q\phi}$, where we defined $q = 1/b - b$ and $Q = 1/b + b$. This theory describes a non-critical string with a matter central charge $c = 1 - 6q^2 < 1$ (see [20]). We also need to add the Liouville term $\mu \int dz^2 e^{2b\phi}$ to regulate the strongly coupled region as usual. The matrix model dual of this $c < 1$ string was constructed in [20] and it is given by the action (2.1) with the constant string coupling g_s replaced with the time-dependent one $g_s \cdot e^{qt}$. In the final stage, we will take the limit $b \to 1$, which is equivalent to the $c = 1$ string. The analogous discussion in type 0 string can be done in the same way and we will omit it.

Since only the time coordinate X^0 obeys the Neumann boundary condition, there are only two possible choices (in-coming and out-going states) for the physical vertex operator. They are given by

$$V_- = e^{-bX^0}\,, \qquad \text{and} \qquad V_+ = e^{X^0/b}\,. \tag{3.3}$$

Now let us consider the on-shell scattering amplitudes by using these vertex operators. A crucial condition is the energy conservation and we have to check this first. Notice that there is no Liouville term in the time direction. Thus we get the constraint from the energy conservation (this is more obvious when we assume the euclidean time)

$$-N_- b + \frac{N_+}{b} = \left(\frac{1}{b} - b\right)\chi\,, \tag{3.4}$$

where $N_\pm(\geq 0)$ denotes the number of the insertions of $V_\pm$, and χ is the Euler number.

Let us assume that b^2 is an irrational number. We can rewrite (3.4) as

$$N_+ - N_- b^2 = \chi - \chi b^2\,. \tag{3.5}$$

This immediately requires

$$N_+ = N_- = \chi\,. \tag{3.6}$$

[7] In this case the amplitude is proportional to $\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t-1)}$ + (2 permutations), when we set $\alpha' = 1$. The kinematical constraint is $s = -2, t = u = 0$. We can check this is vanishing in the almost same way as in the bosonic string.

When we are interested in the disk amplitudes ($\chi = 1$), only two point function $N_+ = N_- = 1$ satisfies this condition. Also for the cylinder amplitudes ($\chi = 0$) only zero point function is non-trivial. All higher open string amplitudes should be zero since they do not satisfy the energy conservation. Thus $N \geq 3$ particle on-shell scatterings are all zero at any order of perturbation theory.

Next we require that the amplitudes should be continuous with respect to the parameter b. This is also expected from its matrix model dual [20]. Then we can obtain the same conclusion for any values of b. Finally, taking the limit $b = 1$, we have the desired conclusion that the $N \geq 3$ particle on-shell scatterings are all zero at any order of perturbation theory in the 2d string theory.

If we consider a theory where we cannot change the value of b continuously, it is possible that some of non-trivial amplitudes are non-zero only when b^2 is a rational number (see (3.4)). This seems to correspond to the minimal string case [28] (see also e.g. [29] for modern viewpoints). Indeed, in minimal models it is impossible to change the values of b continuously as the (p, q) minimal model corresponds to the value $b = \sqrt{p/q}$.

In this way, we have seen that open string tachyons for unstable D-branes have no interaction with themselves in on-shell amplitudes. This conclusion strongly supports our claim in section 2 that the open string field theory is equivalent to the free quadratic action via a smooth field redefinition. This is because any smooth field redefinition does not change S-matrices.

3.3 field redefinition between BSFT and matrix model

Here we would like to find a field redefinition which relates the BSFT action to the $c = 1$ matrix model one up to the cubic order. We will show that it is indeed smooth one.[8] We neglect open string modes other than the open string tachyon into the BSFT action of the ZZ-brane since they are expected to play no important role in BSFT at the classical level.

We consider the open string theory on an unstable D0-brane in 2d string theory. The important point is that this theory is again equivalent to a open string theory in one dimension because open string modes on a ZZ-brane do not excite the Liouville field ϕ. Thus the BSFT action for a D0-brane in 2d string is the same as the zero dimensional truncation of the BSFT action in the 26 dimensional critical string or, if we consider type 0 string theory, the one in the 10 dimensional critical string theory. Then we can also generalize the same result to the case of multiple D0-branes because we can always diagonalize the matrix T by the $\mathrm{U}(M)$ gauge fixing, leading to essentially the abelian action.

The BSFT action in bosonic string up to the cubic order was computed in [30, 31] as follows after the Fourier transformation

$$S_{BSFT} = \frac{1}{g_s}\Big[2\pi\delta(0) - \frac{1}{2}\int dk(2\pi)A(k)T(k)T(-k) + \\ + \frac{1}{6}\int dk_1 dk_2 (2\pi) B(k_1, k_2, -k_1 - k_2)T(k_1)T(k_2)T(-k_1 - k_2) + \cdots\Big], \tag{3.7}$$

[8] Here smooth means smooth near the on-shell momentum.

where we introduced two functions $A(k)$ and $B(k_1, k_2, k_3)$ defined by

$$\begin{aligned} A(k) &= \frac{\Gamma(2-2k^2)}{\Gamma(1-k^2)^2} = 2\frac{4^{-k^2}}{\sqrt{\pi}}\frac{\Gamma(\frac{3}{2}-k^2)}{\Gamma(1-k^2)}\,, \\ B(k_1,k_2,k_3) &= 2(1+\sum_{i<j} k_i k_j) I(k_1,k_2,k_3) - (J(k_1,k_2,k_3) + \text{cyclic permutations})\,. \end{aligned}$$

Here $2\pi\delta(0) = \int dt$ and we defined

$$\begin{aligned} 2\left(1+\sum_{i<j} k_i k_j\right) I(k_1,k_2,k_3) &= \frac{4^{\sum \alpha_i}}{4\pi}\frac{\Gamma(\sum \alpha_i + 1/2)}{\sqrt{\pi}}\frac{\Pi_{i=1}^3 \Gamma(\alpha_i)}{\Pi_{j<k}^3 \Gamma(\alpha_j+\alpha_k)}\,, \\ J(k_1,k_2,k_3) &= \frac{4^{\sum \alpha_i}}{4\pi}\frac{\Gamma(\alpha_2+\alpha_3+1/2)}{\Gamma(\alpha_1+1/2)}\frac{\Gamma(\alpha_1)}{\Gamma(\alpha_2+\alpha_3)}\,, \end{aligned} \tag{3.8}$$

where $\alpha_1 = k_2 k_3 + 1/2, \alpha_2 = k_1 k_3 + 1/2, \alpha_3 = k_1 k_2 + 1/2$, which becomes $\alpha_1 = \alpha_2 = \alpha_3 = 0$ when all three momenta are on-shell.

Then we can find a field redefinition into the $c = 1$ matrix model

$$T(k) = \alpha(k)\Phi(k) + \int dk' \beta(-k,k')\Phi(k-k')\Phi(k') + \cdots\,, \tag{3.9}$$

where we defined

$$\alpha(k) = \sqrt{\frac{1-k^2}{A(k)}}\,, \tag{3.10}$$

$$\beta(k_1,k_2) = \left[\frac{\alpha(k_2)\alpha(k_3)}{2A(k_1)}\left(\frac{2}{3}(1+\sum_{i<j} k_i k_j) I(k_1,k_2,k_3) - J(k_1,k_2,k_3)\right)\right]_{k_3=-k_1-k_2}. \tag{3.11}$$

By this field redefinition, the BSFT action (3.7) is actually mapped to the matrix model action (2.1). Here we have used the fact that the cyclic permutations of J give the same contribution in (3.7). It is clear that the factor $\alpha(k)$ is non-singular since $A(k)$ has a zero at the on-shell point $k^2 = 1$. Next, another one $\beta(k_1, k_2)$ looks divergent at $k_1^2 = 1$ since it includes the factor $A(k_1)^{-1}$. However, we can see that it is successfully canceled with the zero of the other factor expressed by the $1/\Gamma(\alpha_2 + \alpha_3) \sim (k_1^2 - 1)$ in (3.8).[9] Thus we can conclude this field redefinition is non-singular.

One might think this can also be applies to higher dimensional D-branes. However, this is not true. As pointed out in [25], near the on-shell point $|\alpha_i| << 1$, we find

$$\begin{aligned} B(k_1,k_2,k_3) &= \frac{1}{4\pi}\left(\frac{\Pi_{j<k}^3 \alpha_j + \alpha_k}{\Pi_{i=1}^3 \alpha_i} - \frac{\alpha_1+\alpha_2}{\alpha_3} - \frac{\alpha_2+\alpha_3}{\alpha_1} - \frac{\alpha_3+\alpha_1}{\alpha_2}\right) + \mathcal{O}(\alpha_i) \\ &= \frac{1}{2\pi} + \mathcal{O}(\alpha_i)\,, \end{aligned}$$

[9] In other words, the cubic term is proportional to the on-shell factor $k^2 - 1$. This property is very special for the BSFT action and also important for a consistency check of the BSFT as a string field theory (the vanishing of the 1-particle reducible diagram) [31, 25].

where we assumed all α_i are of the same order. This $\frac{1}{2\pi}$ is the correct S-matrix for three open string tachyons. Thus the BSFT reproduces the on-shell interaction term for three open string tachyons in a highly nontrivial way. Therefore, the field redefinition (3.11) becomes singular when k is on-shell because the momenta k_1, k_2 and $-k_1 - k_2$ can be on-shell at the same time. In one-dimension, it is impossible that all three momenta are on-shell due to the momentum conservation. This is crucial for the smoothness of the field redefinition.

We note that this field redefinition is also consistent with the rolling tachyon solution [22]. The BCFT solution $T = \widetilde{\lambda}\cosh(t)$ should correspond to the solution $\Phi = \sin(\pi\widetilde{\lambda})\cosh(t)$ in the matrix model by matching the energy [7, 9]. For the profile $\Phi = \sin(\pi\widetilde{\lambda})\cosh(t)$, we obtain $\alpha(k) = 1$, and the quadratic term in (3.11) vanishes since $\beta(-k, k')$ is zero for $k^2 = 1$. Thus we find $T = \Phi + \mathcal{O}(\widetilde{\lambda}^3)$ which is actually consistent with the energy matching.[10] To consider higher order in $\widetilde{\lambda}$ is an interesting question. We can instead consider a constant tachyon. Note that the field $\tilde{T}$ in which the tachyon potential is the standard BSFT potential $(1+\tilde{T})e^{-\tilde{T}}$ is mapped to T by $T = 1 - e^{-\tilde{T}}$ [30]. For $\tilde{T} = c$, where c is a constant, the field redefinition (3.11) gives $\Phi = c - \frac{1}{3}c^2 + \mathcal{O}(c^3)$. The BSFT action evaluated for this T is $(1+\tilde{T})e^{-\tilde{T}} = 1 - \frac{1}{2}c^2 + \frac{1}{3}c^3 + \mathcal{O}(c^4)$, which is indeed equals to $1 - \frac{1}{2}\Phi^2$. Thus we find the cubic term in the usual BSFT tachyon potential is eliminated by the field redefinition explicitly.

Finally, we will briefly comment on a subtlety related to the rolling tachyon solution in the BSFT. In the BSFT action (3.7) we drop total divergence terms, for example a term linear in T, although the total divergence terms in the lagrangian can contribute to the partition function Z for the rolling tachyon solution. (This is because the partial integration can not be done due to the divergence of the integration of time.) Since the pressure and the Z are same for the Dp-brane, the linear term indeed appeared in the pressure of the rolling tachyon [22]. On the other hand, the energy and pressure are local quantities, for which the total divergence terms can not be contributed.[11] Thus it seems something wrong with the BSFT action (3.7). However, we can assume the term linear in T which contains, for example, the world volume scalar curvature for computing the energy-momentum tensor. In the flat space, such term does not contribute to the BSFT action and the energy but changes the pressure of the homogeneous rolling tachyon solution. (In the Noether current method, if it would be available, this change of the energy-momentum tensor may correspond to the usual ambiguity of the definition of the Noether current.) Therefore it is reasonable to drop the total divergence term in (3.7).

[10] We note that in BCFT it is trivial to obtain the rolling tachyon solution with $\sinh(t)$ from the one with $\cosh(t)$ by the transformation $x^0 \to x^0 + i\pi, \widetilde{\lambda} \to i\widetilde{\lambda}$ in the superstring case [22]. Thus if we assume the analyticity and the consistency for the $\cosh(t)$ solution, it is obvious that the field redefinition is also consistent for the $\sinh(t)$ solution.

[11] Here we defined the energy and the pressure through the coupling to the metric as in [32]. In [32] the zero mode integral is factorized and covariantized by multiplying the volume factor $\sqrt{g}$. More properly, we should multiply $1/g_s$ instead of $\sqrt{g}$. Since the closed string vertex operators for graviton and dilaton are mixed in the Einstein flame, we should add the contribution from the differentiation of the dilaton other than the graviton in order to obtain the energy-momentum tensor. Then the result is same by multiplying the $\sqrt{g}$ term except an overall constant.

4. Scaling limit (closed string decoupling limit)

4.1 Large N scaling limit

As is well-known, the $c = 1$ matrix model originally was come up with as a method of discretization of the world-sheet [5]. In that context, we start with a matrix model with a cubic Φ^3 or quartic interaction Φ^4 and then take the so called double scaling limit. This limit infinitely magnifies the region near the top of the potential and thus the model becomes equivalent to the quadratic matrix model, i.e. (2.1).

One may wonder if this kind of limiting procedure may be relevant to the present interpretation from the viewpoint of open-closed duality using unstable D0-branes. The open string tachyon potentials in string field theories typically include such a cubic T^3 or quartic term T^4. Thus one may speculate that the string field theory action originally includes such an interaction term and it will be simplified after a limiting process such as the decoupling limit.

However, this naive scenario is not actually relevant in our open-closed duality as long as we consider unstable D0-branes in the 2d string theory defined by the conventional Liouville theory. This is because the BSFT action already itself is equivalent to the quadratic action via a field redefinition as we have shown before. Thus we do not have to take any scaling limit to make the action simple. In this interpretation, we inevitably encounter the constraint[12] (2.2) when we try to relate the tachyon T to the matrix Φ by the field redefinition because the energy of the classical solutions are bounded from below. This classical constraint can be interpreted as the infinitely many fermions filling the energy level below $E = -\mu$. Of course, this coincides with the matrix model description of the ZZ-branes proposed in [7].

Although this argument for the existence of the infinitely many fermions, which is responsible for the closed string excitations, is obviously very natural, it is highly nontrivial to show it from the BSFT point of view. One way to see this is to consider a scaling limit of the tachyon action such that we can forget about the energy bound. We will see also that the closed string is decoupled from the D-brane action in this limit.

First, consider putting N ZZ-branes in the 2d string background specified by the coupling constant $g_s = 1/\mu$. The energy level of the ground state of the fermion action is a function of N and g_s denoted by $\tilde{\mu}(N, g_s)$. Then we take the scaling limit $N \to \infty$ and $g_s = 1/\mu \to 0$ with $\tilde{\mu}(N, g_s)$ fixed finite. In the limit, we can forget about the energy bound because of $\mu \to \infty$ and the BSFT action is simply given by the double scaled matrix model action with the Fermi level $-\tilde{\mu}$. This also implies the decoupling of the closed string since the the closed string excitations are the fluctuations of the fermions with the energy below $-\mu \to -\infty$, which cannot interact with the finite energy fermions.

[12] It seems that the "phase space" in the original variable is infinite dimensional due to the terms including higher derivatives and the constraint which corresponds to (2.2) is ambiguous. However, it is two dimensional according to our claim which implies that the BSFT action depends on only two independent (non-local) functions of the $\frac{d^nT}{dt^n}$. Thus the constraint in the "phase space" is unambiguous.

Therefore, we can obtain the matrix model action from the BSFT action of ZZ-branes by taking this rather trivial scaling limit. Using the known relation between the matrix model and 2d string, it is this scaling limit that we should take in order to describe the 2d string non-perturbatively by the theory on the D-branes.

It may be also useful to compare our interpretation with the AdS/CFT correspondence [33], which is the most well-studied example of open-closed duality. It sounds reasonable to relate the scaling limit explained just before with the near-horizon limit in the AdS/CFT. However, we should note that in our example of 2d string theory, taking the limit only changes the background trivially. We can more properly say that it is an analogue of changing radius of the AdS background by adding D3-branes as can be imagined from the matrix model with a harmonic potential [34] for half BPS states in $N = 4$ super Yang-Mills theory.

In this way we have observed that the standard double scaling limit in $c = 1$ matrix model is not relevant in our analysis of open-closed duality. Nevertheless, there is an open possibility that there exists an unknown (possibly highly curved) background in 2d string theory or critical string theory which is exactly described by the matrix model before taking the double scaling limit. If this is true, the conventional 2d string theory can be thought of as a 'near horizon limit' of this new string theory in the true sense.

4.2 Back-reaction of ZZ-brane

To check previous holographic interpretation of 2d string theory in terms of ZZ-branes, let us compute[13] the back-reaction[14] when we put a ZZ-brane just on top of the Fermi level. If our interpretation is correct it should shift the value of the Fermi level by $\delta\mu = \frac{1}{\rho(\mu)} \sim \frac{1}{\log\mu}$, where $\rho(\mu)$ is the density of fermionic state. Here we use the same notation and convention of (bosonic) Liouville theory as those in section 3.1.2.

In the quantum Liouville theory, the wave function $\psi(\phi)$ of zero-mode is given by

$$\psi(\phi) = e^{iP\phi} + S(P)e^{-iP\phi}\,, \tag{4.1}$$

where the phase

$$S(P) = -e^{2i\delta(P)} = -\mu^{-iP/2}\frac{\Gamma(iP)}{\Gamma(-iP)}\,, \tag{4.2}$$

represents the reflection due to the Liouville potential[15]. When we consider the corresponding operator, we have to multiply the Liouville dressing $e^{\frac{Q}{2}\phi}$.

The bulk one point function on the disk for a ZZ-brane can be computed as [6] (setting $b = 1$)

$$\Psi(P) = \langle e^{i(P+\frac{Q}{2})\phi}\rangle = \frac{2}{\sqrt{\pi}}i\sinh(\pi P)\mu^{-iP/2}\frac{\Gamma(iP)}{\Gamma(-iP)} = \frac{2}{\sqrt{\pi}}i\sinh(\pi P)e^{i\delta(P)}\,. \tag{4.3}$$

[13] One of the authors TT is very grateful to Andrew Strominger for very useful discussions on which some of the results in this subsection are based.

[14] See also [4, 35] for related computations.

[15] Here μ denotes the renormalized cosmological constant $\mu = \pi\gamma(b^2)\mu_0$.

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In other worlds, the boundary state of ZZ brane can be written as follows

$$|B\rangle_{ZZ} = \int_{-\infty}^{\infty} dP\ \Psi(-P)|P\rangle \tag{4.4}$$

$$= \int_{-\infty}^{\infty} dP\ \Psi(-P)(e^{iP\hat{\phi}} + S(P)e^{-iP\hat{\phi}})|0\rangle\,. \tag{4.5}$$

The fermion put just on top of the Fermi level is equivalent to the unstable D0-brane decaying with the maximal amount of the rolling tachyon [22], so called $\lambda = 1/2$ brane. This brane corresponds to an array of D-instantons at $t = 2\pi i(n + \frac{1}{2})$ ($n \in \mathbf{Z}$) along the imaginary time [22, 36]. The closed string field $|C\rangle$ (only massless 'tachyon' exists in our two dimensional string) is induced by the presence of these D-branes following $|C\rangle = \frac{1}{L_0+\bar{L}_0}|B\rangle$ at the linear level.

We find the massless scalar field $\varphi(t,\phi)$ sourced by the $\lambda = 1/2$ brane

$$\varphi(t,\phi) = \int dEdP \frac{i\sinh(\pi P)e^{-i\delta(P)}}{E^2 - P^2 - i\epsilon} \cdot e^{iEt} \cdot \left(\sum_{n=-\infty}^{n=\infty} e^{iE(n+\frac{1}{2})a}\right) \cdot (e^{iP\phi} + S(P)e^{-iP\phi})\,, \tag{4.6}$$

where we employed the Feynman propagator. The constant a should finally be equal to i. To perform a sensible analytical continuation, we can first rotate as $E = iP_0, t = -ix_0$. Next we do the integration and finally re-rotate $x_0 = it$ again. Then we get the expression

$$\varphi(t,\phi) = i\int_{-\infty}^{\infty} \frac{\cos(Pt)}{P}(e^{-i\delta(P)}e^{ip\phi} - e^{i\delta(P)}e^{-ip\phi})\,. \tag{4.7}$$

Using the formula of Bessel function (for $y > 0$)

$$J_0(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} ds \frac{\Gamma(-is)}{\Gamma(1+is)}(\frac{y}{2})^{2is}, \tag{4.8}$$

we get

$$\varphi(t,\phi) = -4\pi\left(J_0(2\mu^{\frac{1}{4}}e^{\frac{\phi+t}{2}}) + J_0(2\mu^{\frac{1}{4}}e^{\frac{\phi-t}{2}})\right). \tag{4.9}$$

For example, in the weak coupling region $\phi \to -\infty$, the expression is approximated by

$$\varphi(t,\phi) \sim -4\pi\left(2 - \sqrt{\mu}e^{\phi+t} - \sqrt{\mu}e^{\phi-t}\right). \tag{4.10}$$

Thus at an early time we find an incoming wave (toward the strongly coupled region), while at late time the outgoing wave is dominant. This is consistent with the fact that the brane creation and annihilation take place in the strongly coupled region ($\phi \to \infty$), where the ZZ-brane is located. The constant part shifts the cosmological constant μ by $\delta\mu \sim \frac{1}{\log\mu}$, as can be understood from its back-reaction to the Liouville term $\mu\phi e^{2\phi}$ at the typical region $\phi \sim -\frac{1}{2}\log\mu$. This is exactly what we would like to show.

5. Implications for the BSFT in higher dimension

It will be worth considering how our results in 2d string theory are helpful to the understanding of the BSFT in higher dimension. In general, the BSFT action includes infinitely many higher derivative terms. This leads to many complications as those in the initial value problem, the definition of its phase space, the quantization of the theory and so on. In the 2d string case, however, we have shown that the action can be greatly simplified via a field redefinition into the quadratic one. Then we can quantize everything very easily in the latter formulation. Notice that even though this model is so simple and free, it includes non-trivial dynamics like closed sting S-matrices. Thus the correct path-integral measure is given by the standard measure for the matrix model variable if our claim is correct and the BSFT is actually a string field theory. This implies that the path-integral measure in the BSFT variable should be complicated because of the jacobian of the field redefinition. We might be able to find it from the contribution from some fields other than tachyon.

A similar simplification may occur in the higher dimensional BSFT. This will be very useful for further analysis of BSFT in future. For example, this may shed some light on the problem of quantizing BSFT, which is very difficult at this point (e.g. see [37]). Although it is not an easy task to pursue this direction, we can at least notice the following interesting property of the BSFT action in the ordinary 26 dimensional bosonic string or 10 dimensional type II or type0 string theory. If we set all fields in the BSFT other than the tachyon field T to zero, then the unstable D0-brane action is exactly the same as the one we studied in this paper. Thus this truncated action is equivalent to the quadratic action. Classically it is meaningful to consider the subset of the configuration space such that the massless and massive fields vanish and solve the equations of motion. Quantum mechanically, it is not allowed to fix some fields to some values by hand in general. Therefore it is very important to include the effects of, in particular, the massless fields to the tachyon action. Though this is very difficult problem as stated in the introduction, we hope we will find some simple way to incorporate such effects into the matrix model action in near future.

Another interesting lesson is for the problem of the restriction of the momentum [25]. The function $A(k)$ is divergent at $k^2 = 2, 3, \cdots$ and becomes negative, for example $2 < k^2 < 5/2$. Thus the $T(k)$ is not a good coordinate for $2 < k^2$. However, in the matrix model side, there is no momentum restriction. This suggests that we can extend the BSFT variable $T(k)$ to $2 < k^2$ by proper field redefinition around $k^2 = 2$ although $T(k)$ is not valid for $2 < k^2$ even in the higher dimensional BSFT.

6. Conclusion

In this paper we have argued that the BSFT action for unstable D0-branes in 2d string theory is classically equivalent to the double scaled $c = 1$ matrix model action (i.e. quadratic action) via a field redefinition. This gives a simple derivation of the $c = 1$ matrix model action from the open string field theory of unstable D0-branes. We have given strong evidences for this claim. First, we showed that the S-matrices of open string tachyons on the D0-branes are trivial. Also we have explicitly constructed a smooth field redefinition up to the cubic term in the BSFT action. To understand this equivalence from the viewpoint of open-closed duality, we also pointed out a scaling limit, which can be regarded as a closed string decoupling limit. Finally we discussed how our results in 2d string can be generalized to the BSFT in higher dimension.

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An interesting point in the equivalence is that the field redefinition is smooth, but non-local in time. This can be seen from the fact that the field redefinition (3.11) contains non-polynomial functions of the time derivatives. Although we do not have any physical interpretation of the non-locality, it may be related to some interesting phenomena and worth investigating.

There are several interesting future directions. In this paper we only discussed BSFT among open string field theories. It would be intriguing to study what is the precise relation between $c = 1$ matrix model and the cubic string field theory[16] [41]. It is also curious to see if the similar arguments in our paper can be applied to other matrix models such as those for the minimal string.

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